

## MATH

### UNIT - 1

#### Analysis: Elementary set theory, Sets:

Sets and their representations. Empty set, Finite & Infinite sets, Equal sets. Subsets, Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set, Properties of complement sets.

Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.

#### Sequence and series:

Sequence and Series, Arithmetic Progression (A.P), Arithmetic Mean (A.M), Geometric Progression (G.P), general term of a G.P, sum of  $n$  terms of a G.P. Arithmetic and Geometric series, infinite G.P. and its sum. Geometric mean (G.M), relation between A.M and G.M, Sum to  $n$  term of the special series  $\sum n$ ,  $\sum n^2$  and  $\sum n^3$ . Convergence, Sequences and series of functions, uniform convergence.

#### Binomial Theorem:

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications. Bolzano Weierstrass theorem, Heine Borel theorem.

#### Continuity and Differentiability:

Continuity, uniform continuity. Continuity and Differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit function.

Concepts of exponential and logarithmic functions. Derivatives of  $\log_e x$  and  $e^x$ . Logarithmic differentiation. Derivative of functions expressed in parametric forms.

Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

#### Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents and normal, approximation, maxima and minima. Simple problems (that illustrate basic principles and understanding of the subject as well as real life situations).

#### Limits and Derivatives:

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive idea of limit,  $\limsup$ ,  $\liminf$ . Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Mean value theorem.

Integrals: Integration as Inverse process of differentiation. Integration of a variety of functions by



Substitution, by partial fractions and by parts. Definite integrals as a limit of a sum. Fundamental Theorem of Calculus, Basic properties of definite integrals and evaluation of definite integrals.

#### Applications of the Integrals:

Applications in finding the area under simple curves, especially lines, areas of circles/parabolas/ellipses, area between the two above said curves. Riemann sums and Riemann integral, Improper Integrals.

Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

#### Linear Algebra:

Vector spaces: Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors, scalar triple product.

#### Linear Inequalities:

Linear inequalities, Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables - graphically.

Subspaces, linear dependence, basis, dimension, algebra of linear transformations.

#### Matrices & Determinant:

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists.

Determinant of a square matrix (up to  $3 \times 3$  matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle.

Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equation by examples, solving system of linear equations in two or three variables using inverse of a matrix. Rank and determinant of matrices, linear equations. Eigen values and eigen vectors, Cayley-Hamilton theorem.

Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

#### Straight Lines :

Brief recall of 2-D from earlier classes, Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two-point form, intercept form and normal form, General equation of a line. Equation of family of lines passing

